GCSE Maths – Algebra

Expressions involving Surds and Algebraic Fractions

Notes

WORKSHEET

Expressions involving Surds and Algebraic Fractions

You need to be able to manipulate surd expressions and those studying **higher** GCSE mathematics need to be able to manipulate expressions involving algebraic fractions.

Surds

The following examples demonstrate how to manipulate expressions involving surds.

Example: Simplify the expression $5\sqrt{2} + \sqrt{72}$

1. Reduce all surds in the expression to their simplest form.

   We can **simplify** $\sqrt{72}$ further:

   $$\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2} = 6 \times \sqrt{2} = 6\sqrt{2}$$

2. Simplify the expression using surd rules.

   $$5\sqrt{2} + \sqrt{72} = 5\sqrt{2} + 6\sqrt{2} = \mathbf{11\sqrt{2}}$$

Example: Simplify the expression $10\sqrt{5} + 6\sqrt{5}\,a + \sqrt{45}a$

1. Reduce all surds in the expression to their simplest form.

   We can **simplify** $\sqrt{45}a$ further:

   $$\sqrt{45}\mathrm{a} = \sqrt{9 \times 5}\ \mathrm{a} = \sqrt{9} \times \sqrt{5} \times \mathrm{a} = 3 \times \sqrt{5} \times \mathrm{a} = 3\sqrt{5}\,a$$

2. Simplify the expression using surd rules.

   $$10\sqrt{5} + 6\sqrt{5}a + 3\sqrt{5}a = \mathbf{10\sqrt{5} + 9\sqrt{5}a}$$

Expansion with surds works in the same way as normal expansion.

Example: Expand the expression $(4\sqrt{2} + 5a)(10 + 6\sqrt{2})$

1. Check to see if we can simplify any surds.

   *In this case we **cannot** do further simplification.*

2. We will use the **FOIL** method to expand.

   F: $4\sqrt{2} \times 10 = 40\sqrt{2}$
   O: $4\sqrt{2} \times 6\sqrt{2} = +48$
   I: $+5a \times 10 = +50a$
   L: $+5a \times 6\sqrt{2} = (30\sqrt{2})a$

   This leaves us with: $\mathbf{40\sqrt{2} + 48 + 50a + 30\sqrt{2}\,a}$

   *Although we do technically have two constant terms and two letter terms, this **must not be simplified further** because adding the surd constant to the integer constant will give a less exact solution.*

Factorising surds works in the exact same way as 'normal' factorising.

Example: Factorise the expression $10\sqrt{3}a + \sqrt{48}\,ac$

1. Simplify any surds that can be simplified further.

$$\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}.$$

So,

$$10\sqrt{3}a + \sqrt{48}\,ac = 4\sqrt{3}ac + 10\sqrt{3}\,a$$

2. See if we can factorise any terms out of the expression. We first check to see if they have any **common number factors** and then any **common letters factors.**

*It may look like we have **no common number terms** but as we are dealing with **surds** we can actually take out the surd as a common factor. So, in this case we can factorise out $\sqrt{3}$ and an 'a' term:*

$$4\sqrt{3}ac + 10\sqrt{3}\,a = \boldsymbol{\sqrt{3}a(4c + 10)}$$

3. **Check** if this is the correct answer by expanding out the final answer. If it is correct, we will obtain the original expression.

$$\sqrt{3}a(4c + 10) = 10\sqrt{3}a + 4\sqrt{3}ac$$

The answer is indeed $\boldsymbol{\sqrt{3}a(4c + 10)}$.

Algebraic Fractions (Higher Only)

Algebraic fractions are just like normal fractions, except they contain letters as well as numbers.

Simplifying Algebraic Fractions

- **Cancel any numbers** that are present in the numerator and denominator.
- **Cancel the letters individually** (by dividing the top and bottom of the fraction).

Example: Simplify the expression $\frac{32x^3y^2}{8xy}$

1. Cancel any number terms from the denominator and the numerator.

Divide the numerator and denominator by 8:

$$\frac{32x^3y^2}{8xy} = \frac{4x^3y^2}{xy}$$

2. Cancel any letter terms from the denominator and the numerator.

Divide the numerator and denominator by x and then y:

$$\frac{4x^3y^2}{xy} = \frac{4x^2y^2}{y} = \boldsymbol{4x^2y}$$

Example: Simplify the expression $\frac{x^2-16}{x^2+5x+4}$

It might be tempting to try and simplify the numbers **BUT we cannot do this** as **they are not multiplied with any terms;** they are added or subtracted onto further terms! Then it seems that we can cancel x terms, but we **cannot** do this either!

1. **Factorise** any quadratic expressions if possible.

$$\frac{x^2-16}{x^2+5x+4} = \frac{(x+4)(x-4)}{(x+1)(x+4)}$$

If stuck: *see notes on 'Factorising Linear and Quadratic Expressions'.*

2. Cancel any factorised terms which appear in the numerator and the denominator.

We can ***simplify the fraction*** *as both the numerator and denominator have an* $(x+4)$ *term. We can cancel this by dividing the numerator and denominator by* $(x+4)$*:*

$$\frac{x^2-16}{x^2+5x+4} = \frac{(x+4)(x-4)}{(x+1)(x+4)} = \boldsymbol{\frac{x-4}{x+1}}$$

Multiplying and Dividing Algebraic Fractions

Multiplying and dividing algebraic fractions follows the same rules of normal fractions.

- **For multiplication:** Perform cancellations, and then separately multiply the numerators together and the denominators together.
- **For division:** Flip the second fraction and multiply this with the first fraction, following the rules of fraction multiplication stated above.

Example: Simplify the expression $\frac{x^2}{8} \times \frac{4}{x+3}$

1. Separately multiply the numerators together and the denominators together.

$$\frac{x^2}{8} \times \frac{4}{x+3} = \frac{4x^2}{8(x+3)}$$

2. Look for any terms which might cancel.

In this case we can cancel the number terms, by dividing both fractions by 4:

$$\frac{x^2}{8} \times \frac{4}{x+3} = \frac{4x^2}{8(x+3)} = \frac{x^2}{2(x+3)} = \boldsymbol{\frac{x^2}{2x+6}}$$

Example: Simplify the expression $\frac{3}{x} \div \frac{x^5}{4}$

1. **Flip the second fraction** and then **multiply them together**

$$\frac{3}{x} \div \frac{x^5}{4} = \frac{3}{x} \times \frac{4}{x^5}$$

2. Before multiplying, first try to see if you can **cancel** anything. This will simplify subsequent calculations.

   In this case we cannot.

3. Separately **multiply the numerators** together and the **denominators** together.

$$\frac{3}{x} \div \frac{x^5}{4} = \frac{3}{x} \times \frac{4}{x^5} = \frac{3 \times 4}{x \times x^5} = \boldsymbol{\frac{12}{x^6}}$$

Adding and Subtracting Algebraic Fractions

Adding and subtracting algebraic fractions follows the same rules of normal fractions.

1. Work out what the **common denominator will be**.
2. Write each fraction with the same common denominator.
3. Add or subtract **only** the **numerators**.

Example: Write $\frac{4}{(x+3)} + \frac{2}{(x-2)}$ as a single fraction in its simplest form

1. Work out what the common denominator will be. A common denominator can be found by multiplying the two denominators together

   *A **common denominator** in this case can be* $(x+3)(x-2)$.

2. Write each fraction with the same common denominator.

   We need to multiply the denominator and numerator of the first fraction by $(x-2)$ *and the denominator and numerator of the second fraction by* $(x+3)$*:*

$$\frac{4}{(x+3)} + \frac{2}{(x-2)} = \frac{4(x-2)}{(x+3)(x-2)} + \frac{2(x+3)}{(x-2)(x+3)} = \frac{4(x-2)+2(x+3)}{(x+3)(x-2)}$$

3. **Simplify** the numerator by **expanding** and **collecting** the **like terms**.

   Collecting like terms leads to $6x-2$. *We can also factorise this and write it as* $2(3x-1)$.

$$\frac{4(x-2)+2(x+3)}{(x+3)(x-2)} = \frac{4x-8+2x+6}{(x+3)(x-2)} = \frac{6x-2}{(x+3)(x-2)} = \boldsymbol{\frac{2(3x-1)}{(x+3)(x-2)}}$$

Expressions involving Surds and Algebraic Fractions – Practice Questions

1. Simplify the following:

   a) $\sqrt{78}d + 5\sqrt{13} - 19\sqrt{13}d$
   b) $6\sqrt{10}p - 9\sqrt{10} + \sqrt{40}p$

2. Expand the following:

   a) $(9c + 5)(7\sqrt{3} + 4)$
   b) $(16 + 11\sqrt{13})(-8 + 4p)$

3. Factorise the following:

   a) $\sqrt{52}e + 3\sqrt{13}e$
   b) $12\sqrt{12}st + \sqrt{12}s$

4. Simplify the following:

   a) $\frac{(x+2)(x+1)}{x^2+5x+6}$
   b) $\frac{y^2+2y-3}{(y+3)(y+4)}$

5. Simplify the following:

   a) $\frac{(x+3)}{3} \times \frac{6}{(x+2)}$
   b) $\frac{8}{2(y+2)} \times \frac{(y+2)}{(y+8)}$

6. Simplify the following:

   a) $\frac{3}{x} \div \frac{x}{4}$
   b) $\frac{17}{4b^4} \div \frac{3b^4}{8}$

7. Write the following expressions as single fractions.

   a) $\frac{3}{(x+1)} + \frac{8}{(x+7)}$
   b) $\frac{7}{(a+5)} + \frac{8}{(a-3)}$

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.